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SHEAR FORCE EFFECT WITH MULTI-LINEAR HARDENING BEHAVIOR IN ELASTOPLASTIC ANALYSIS WITH MATHEMATICAL PROGRAMMING

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Abstract: *This work deals with limit elastoplastic analysis of steel structures in the framework of mathematical programming. The maximum load carrying capacity of the structure is determined by solving an optimization problem with linear equilibrium, compatibility and yield constraints together with a complementarity constraint that is treated using a penalty function method. Incorporation of shear force effect determines a nonlinear 3D yield surface that is linearized appropriately with a set of planes. Moreover, a cone identification approach is adopted and thus, yield and complementarity conditions are formed only for the specific targeted yield hyperplane for every stress vector at the current loading state. In addition, isotropic hardening/softening cross-sectional behavior is embedded via a linear multi-segment approximation without affecting the size of the problem. The entire formulation succeeds in reducing substantially the size of yield, hardening and complementarity conditions which become independent of the discretization, enabling the solution of large scale problems. Numerical results are presented that verify the validity of the proposed method and highlight the role of shear force effect.*

1 INTRODUCTION

The ultimate load-carrying capacity of the structure is dealt in the framework of mathematical programming as an optimization problem based on piecewise linear constitutive relations following associated flow rules. This formulation, pioneered by Maier et al. [1-4], constitutes a combination of limit and deformation analysis under holonomic or nonholonomic assumption. This means that the maximum load factor is assessed at the final stage of collapse under equilibrium, yield and deformation i.e. compatibility constraints.

A variety of alternative mathematical programming procedures such as iterative Linear Programming, Quadratic Programming, Restricted Basis Linear Programming, Parametric Linear Complementarity and Parametric Quadratic Programming procedures have been applied for elastoplastic analysis of structures [5,6]. The recent development of mathematical programming algorithms appropriate for Mathematical Programming with Equilibrium Constraints (MPEC) problems [7] has extended the potential of the proposed methods for structural analysis for both holonomic and nonholonomic assumptions [8-13].

The aim of this work is to examine the shear force effect on the ultimate load carrying capacity of frame structures. Thus, the adopted yield criterion accounts for the axial-shear force-bending moment interaction and the proposed formulation is applied for rigid-perfectly plastic and isotropic hardening behavior. Moreover, a threefold simplification is proposed concerning the evaluation of strength reserves, the direct evaluation of the hardening/softening response and the complementarity condition. The main step towards these goals is the identification of the particular cone in which the stress vector resides, determined at every loading stage, for every stress vector, during the optimization process. Based on this information both the reserves and the complementarity condition are solely evaluated for this particular cone. Furthermore, detection of the hardening/softening segment at which a critical cross-section is stressed, significantly simplifies the incorporation of multi-linear hardening law into the problem.

The organization of the paper is as follows. First, the governing relations of holonomic elastoplastic problem based on equilibrium, kinematical and constitutive relations are summarized. Then, the formulation of the limit analysis as a MPEC problem is presented incorporating the axial-shear force-bending moment interaction. Subsequently a numerical example of steel frame is presented that illustrates the applicability of the proposed method and the role of shear force effect on the maximum load factor.

2 FORMULATION OF THE PROBLEM

The formulation of the holonomic problem is based on several assumptions. First, plane frames are considered to consist of straight prismatic elements subjected herein only to nodal loading for reasons of simplicity. Moreover, frame displacements are assumed small enough so that the equilibrium equations refer to the initial undeformed configuration. In addition plastic hinges are considered formed only at critical sections, whereas the remaining parts behave elastically. The local nonlinear behavior of critical sections is described by a piecewise linear model and yield functions are linearized appropriately. Furthermore, under monotonically increasing external loading, local unloading if happens, is assumed reversible. Thus a holonomic i.e. path-independent structural behavior is adopted. In the following, the aforementioned relations that govern the elastoplastic behavior are discussed analytically. It is noted that n denotes the number of elements and *nf* the number of degrees of freedom.

2.1 Equilibrium

Each plane beam element develops six stress resultants at its ends, as shown in Figure 1. Three of them are considered as independent, while the remaining dependent actions can be evaluated by applying the three equilibrium equations. Herein, the axial force (s_1^i), bending moment at the start node j (s_2^i) and bending moment at the end node k (s_3^i), as shown in Figure 1, are considered as independent [12,13]. The structural equilibrium relationship for the whole structure is then established as:

$$[B]\cdot\{s\} = a\{f\} + \{f_d\} \tag{1}$$

where B is the (*nf*×*3n*) structural equilibrium matrix, formed by assembling the corresponding element equilibrium matrices, s is a (*3n*×*1*) vector for all primary stress resultants, a is a scalar load factor, f is the (*nf*×*1*) basic monotonically varying nodal forces and f_d is the (*nf*×*1*) fixed nodal vector.

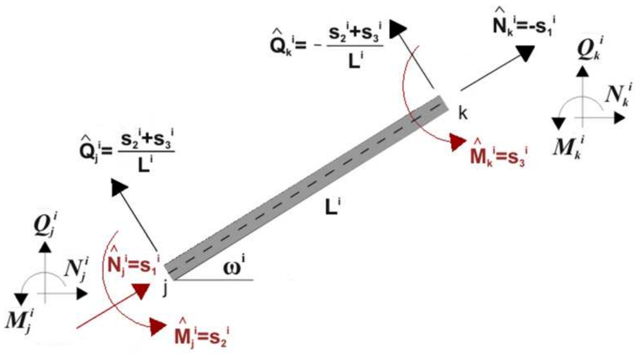

Figure 1: Frame element i with positive stress resultants

2.2 Piecewise linear yield condition

In this work, it is considered that plastic hinges are formed under the combined effect of either axial-shear force-bending moment interaction (NQM interaction) or axial force-bending moment interaction (NM interaction). Thus, for the case of NQM interaction a plastic hinge can be formed at start node j through the combined stresses $(s_1^i,\ \frac{s_2^i+s_3^i}{L^i},\ s_2^i)$ and at end node k through $(-s_1^i,\ -\frac{s_2^i+s_3^i}{L^i},\ s_3^i)$. For the case of NM interaction a plastic hinge can be formed at start node j under the combined stresses (s_1^i, s_2^i) and at end node k due to $(-s_1^i, s_3^i)$. For both cases the nonlinear yield surface is appropriately linearized and thus the yield condition is expressed in the form of linear constraints. The standard formulation involves all the hyperplanes, which increases considerably the number of yield constraints per critical section, complicating the entire formulation also at later stages. It is feasible though, to identify the specific cone in which the stress point resides and consider only one constraint associated to each critical section as potentially active or true active constraint. This consideration reduces the number of yield constraints maintaining only those with physical meaning. Moreover it facilitates the incorporation of multi-linear hardening reducing, in this respect, the complexity of the whole problem.

2.2.1 Identification of the critical cone

The 3D nonlinear yield surface is appropriately linearized with plane triangles. The vertices of each plane triangle together with the origin form one cone (tetrahedron). Each stress point belongs only to one of these cones-tetrahedra and targets at the corresponding plane triangle. Let the stress point be $P=(n,v,m)$ and the tetrahedron have vertices (Figure 2):

$$V_1=(n_1,v_1,m_1),\ V_2=(n_2,v_2,m_2),\ V_3=(n_3,v_3,m_3),\ V_4=(n_4,v_4,m_4) \tag{2}$$

Then the stress point P lays in the tetrahedron if the following all five determinants have the same sign:

$$D_0 = \begin{vmatrix} n_1 & v_1 & m_1 & 1 \\ n_2 & v_2 & m_2 & 1 \\ n_3 & v_3 & m_3 & 1 \\ n_4 & v_4 & m_4 & 1 \end{vmatrix}, D_1 = \begin{vmatrix} n & v & m & 1 \\ n_2 & v_2 & m_2 & 1 \\ n_3 & v_3 & m_3 & 1 \\ n_4 & v_4 & m_4 & 1 \end{vmatrix}, D_2 = \begin{vmatrix} n_1 & v_1 & m_1 & 1 \\ n & v & m & 1 \\ n_3 & v_3 & m_3 & 1 \\ n_4 & v_4 & m_4 & 1 \end{vmatrix}, D_3 = \begin{vmatrix} n_1 & v_1 & m_1 & 1 \\ n_2 & v_2 & m_2 & 1 \\ n & v & m & 1 \\ n_4 & v_4 & m_4 & 1 \end{vmatrix}, D_4 = \begin{vmatrix} n_1 & v_1 & m_1 & 1 \\ n_2 & v_2 & m_2 & 1 \\ n_3 & v_3 & m_3 & 1 \\ n & v & m & 1 \end{vmatrix} \quad (3)$$

The comparison of the signs of D_i and D_0 constitutes a check of whether P and V_i are on the same side of boundary i (namely the boundary that is formed by the three points other than V_i). If P is inside all four boundaries, then it is inside the tetrahedron. If the sign of any D_i differs from that of D_0 then P is outside boundary i , while if any of the determinants $D_i = 0$, then P lies on the boundary i .

The aforementioned procedure is used for the identification of the critical yield plane that corresponds to each cross-section at every loading step. Then the yield condition is formed only for this plane and not for all possible ones, as it is shown in the sequel.

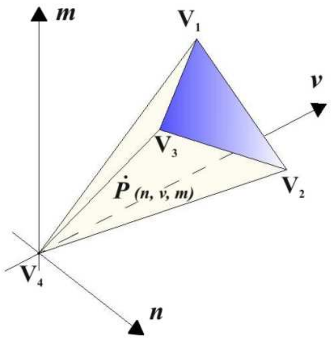

Figure 2: Identification of the critical cone-tetrahedron.

2.2.2 Yield condition

Herein, plastic hinges are considered to develop at the element ends under the axial-shear force-bending moment interaction. Various yield criteria exist in literature for different materials and/or cross sectional shapes; herein the generalized Gendy-Saleeb yield criterion given by the following relation is adopted [14]:

$$\Phi = n^2 + v^2 + \frac{1}{\beta} \cdot m^2 - 1 \quad (4)$$

where $n = s_1^i / s_{1y}^i$, $v = (s_2^i + s_3^i) / L^i \cdot v_y^i$, $m = s_2^i / s_{2y}^i$ or $m = s_3^i / s_{3y}^i$ and s_{1y}^i, v_y^i, s_{2y}^i and s_{3y}^i are the individual axial, shear and moment plastic capacities for the critical cross sections of the elements. It is noted that the above yield relation is valid for both rectangular and wide flange-I cross-sections. The introduced shape dependent parameter β is evaluated for rectangular cross-sections and I-sections respectively using the following relations:

$$\beta = 1 - n^2, \quad \beta = 1 - 1.1|n| \quad (5)$$

In this work, the aforementioned yield criterion is represented by a 3D nonlinear surface that is approximated by using 32 plane triangles (16 for *m>0* and 16 for *m<0*) as shown in Figure 3. This converts the yield condition into a set of linear ones which is advantageous for the mathematical programming formulation of the problem. It is noted that the tessellation of the yield surface is such that the convexity of the yield criterion is maintained. More specifically, the approximation of the 32 plane triangles corresponds to 32 equations for the corresponding planes in $x - y - z$ space, which are of the following form:

$$Ax + By + Cz + D = 0 \quad (6)$$

where A, B, C are the components of the normal vector of the plane and $-D$ the distance of the plane from the origin. At cross-sectional level, using instead of x, y, z the n, v, m variables for the normalized axial, shear force and bending moment respectively and performing an appropriate manipulation, the following forms of these equations are systematically determined for start nodes j with respect to s_1^i, s_2^i, s_3^i as:

$$A_t\cdot\frac{s_1^i}{s_{1y}^i}+B_t\cdot\frac{s_2^i+s_3^i}{L^i\cdot v_y^i}+C_t\cdot\frac{s_2^i}{s_{2y}^i}+D_t=0\Leftrightarrow\frac{A_t}{C_t}\cdot\mu_{21}^i\cdot s_1^i+\frac{B_t}{L^i\cdot C_t}\cdot\mu_{2v}^i\cdot s_2^i+\frac{B_t}{L^i\cdot C_t}\cdot\mu_{2v}^i\cdot s_3^i+s_2^i=-\frac{D_t}{C_t}\cdot s_{2y}^i\Leftrightarrow$$
$$Ac_t\cdot\mu_{21}^i\cdot s_1^i+\left(Bc_t\cdot\mu_{2v}^i+1\right)\cdot s_2^i+Bc_t\cdot\mu_{2v}^i\cdot s_3^i=-Dc_t\cdot s_{2y}^i \tag{7}$$

For end nodes k, it is considered that $\mathrm{N}_k^i=-s_1^i$, $\mathrm{Q}_k^i=-\frac{s_2^i+s_3^i}{L^i}$, $\mathrm{M}_k^i=s_3^i$ and thus the associated plane equations are of the following form:

$$-A_t\cdot\frac{s_1^i}{s_{1y}^i}-B_t\cdot\frac{s_2^i+s_3^i}{L^i\cdot v_y^i}+C_t\cdot\frac{s_3^i}{s_{3y}^i}+D_t=0\Leftrightarrow-\frac{A_t}{C_t}\cdot\mu_{31}^i\cdot s_1^i-\frac{B_t}{L^i\cdot C_t}\cdot\mu_{3v}^i\cdot s_2^i-\frac{B_t}{L^i\cdot C_t}\cdot\mu_{3v}^i\cdot s_3^i+s_3^i=-\frac{D_t}{C_t}\cdot s_{3y}^i\Leftrightarrow$$
$$-Ac_t\cdot\mu_{31}^i\cdot s_1^i-Bc_t\cdot\mu_{3v}^i\cdot s_2^i-\left(Bc_t\cdot\mu_{3v}^i-1\right)\cdot s_3^i=-Dc_t\cdot s_{3y}^i \tag{8}$$

where A_t, B_t, C_t and D_t the coefficients of each plane equation, $Ac_t=A_t/C_t$, $Bc_t=B_t/L^iC_t$, $Dc_t=D_t/C_t$, $\mu_{21}^i=s_{2y}^i/s_{1y}^i$, $\mu_{31}^i=s_{3y}^i/s_{1y}^i$, $\mu_{2v}^i=s_{2y}^i/v_y^i$, $\mu_{3v}^i=s_{3y}^i/v_y^i$, *i=1,…,n* (number of elements) and *t=1,…,16* (m>0). It is noted that for *t=17,..,32* (m<0) the plane equations for each element end are given by:

$$-Ac_t\cdot\mu_{21}^i\cdot s_1^i-\left(Bc_t\cdot\mu_{2v}^i+1\right)\cdot s_2^i-Bc_t\cdot\mu_{2v}^i\cdot s_3^i=Dc_t\cdot s_{2y}^i \tag{9}$$

$$Ac_t\cdot\mu_{31}^i\cdot s_1^i+Bc_t\cdot\mu_{3v}^i\cdot s_2^i+\left(Bc_t\cdot\mu_{3v}^i-1\right)\cdot s_3^i=Dc_t\cdot s_{3y}^i \tag{10}$$

The first part of equations (7) - (10) represents the total stress state of a yielded element end under the effect of NQM interaction. Thus, the coefficients that multiply the variables s_1^i, s_2^i, s_3^i form the *(3×2)* matrix of the scaled normals N^i for each element, while the second part of the equations forms the *(2×1)* vector r^i of the scaled yield limits in terms of bending moment.

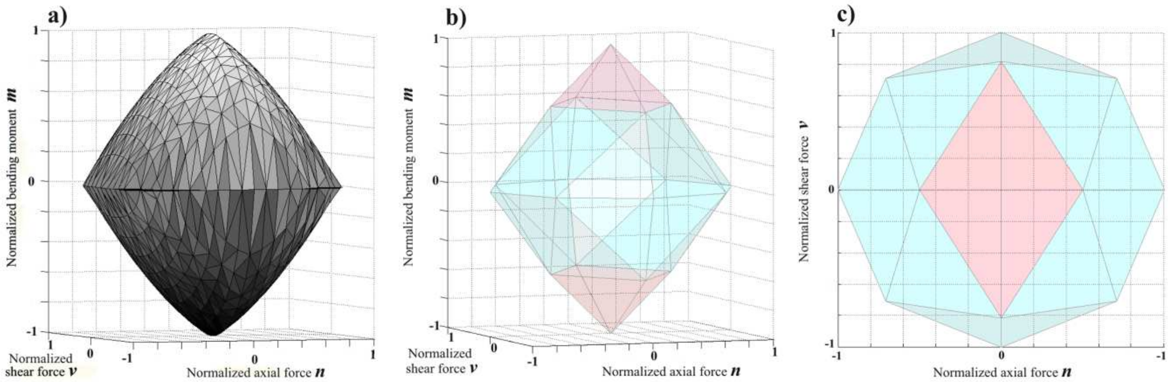

Figure 3: a) Nonlinear Gendy-Saleeb yield criterion, b) Linearized yield criterion and c) Plan view *n-v*.

Incorporating the concept of cone identification and adopting the aforementioned criterion for both types of interaction, the yield condition is formed for the specific hyperplane and not for all possible ones as:

$$\{w\}=-[N]^T\cdot\{s\}+\{r\}\geq 0 \tag{11}$$

where w is the (*2n×1)* vector containing the scaled moment reserves of all stress points, N is the (*3n×2n*) matrix contains all scaled normal vectors of the identified yield hyperplanes and r is the (*2n×1)* vector that includes the yield limits of the critical yield hyperplanes.

It is noted that the cone identification approach reduces the number of yield constraints and converts the yield condition into an independent one from the discretization of the yield surface.

2.2.3 Incorporation of multi-linear isotropic hardening

The linearized yield surface is assumed to follow an isotropic multi-linear hardening rule [13]. At every load instance and for every critical section, the hardening segment ℓ that corresponds to the particular plastic multiplier z is identified. Then, the hardening diagonal matrix H with dimensions (*2n×2n*) and the vector H_0 (*2n×1*) are built for the whole structure. The following relations determine the non-zero entries of the above matrices:

$$H(\kappa,\kappa)=\tau^{\kappa}\cdot h_{\ell}^{\kappa}, \qquad \kappa=1...2n,\ \ell=1...nseg \tag{12}$$

$$H_0(\kappa,1)=\begin{cases} 0, & \kappa=1...2n,\ \ \ell=1 \\ \tau\cdot\sum_{\ell=2}^{nseg}(h_{\ell-1}^{\kappa}-h_{\ell}^{\kappa})\cdot z_{\ell-1}^{\kappa}, & \kappa=1...2n,\ \ \ell=2...nseg \end{cases} \tag{13}$$

where $h_{\ell}^{\kappa}=(\lambda_{\ell}^{\kappa}-\lambda_{\ell-1}^{\kappa})s_{2y}\big/(z_{\ell}^{\kappa}-z_{\ell-1}^{\kappa})$, λ_{ℓ}^{κ} is a scaling factor for yield limit with $\lambda_0^{\kappa}=1$, z_{ℓ}^{κ} is the value of plastic multiplier of the section at the end of segment ℓ and *nseg* is the number of hardening/softening linear segments for the κ^{th} cross section. It is noted that the previous elastic behavior is expressed by the term $N^T s$ and thus $H_0(\kappa,1)=0$. This means that H accounts for the current hardening/softening measure that corresponds to the identified segment, while H_0 is the accumulated total constant previous hardening behavior.

The yield condition for the whole structure is then expressed as:

$$\{w\}=-[N]^T\cdot\{s\}+[H]\cdot\{z\}+[H_o]+\{r\}\geq 0 \tag{14}$$

where z is the ($2n\times1$) vector of all plastic multipliers.

The aforementioned consideration can be expanded for unlimited number of hardening/softening segments without affecting the dimensions of H and H_0 matrices and thus the size of the yield condition. This conception retains the computational simplicity of the cone identification consideration incorporating the multi-linearity of hardening behavior that approaches the real structural behavior.

2.3 Complementarity condition

An additional constraint that regulates elastoplastic behavior of the structure is the complementarity condition given by:

$$\{w\}^T\cdot\{z\}=0, \qquad \{w\}\geq 0, \qquad \{z\}\geq 0 \tag{15}$$

It indicates that when the identified yield function w is activated ($w=0$), the corresponding plastic multiplier z should be greater than zero. Similarly, when the yield hyperplane is inactive $(w>0)$, the corresponding plastic multiplier $z=0$, indicating that no plastic flow occurs. Incorporating the concept of cone identification reduces significantly the number of the implemented equalities since the complementarity condition for each cross-section is expressed with regard to the specific yield hyperplane.

2.4 Compatibility and strain decomposition

Compatibility conditions relate the member deformations q to the nodal displacements u. Since small displacements are considered in this work, the compatibility condition for the whole structure is given by:

$$\{q\}=[B]^T\cdot\{u\} \tag{16}$$

where q is the ($3n\times1$) deformation vector and u is the ($nf\times1$) nodal displacement vector.

The constitutive law that governs the behavior of a generic element is based on deformation decomposition into the elastic and plastic component. For the entire structure this is expressed by:

$$\{q\}=\{e\}+\{p\}=[S]^{-1}\cdot\{s\}+[N]\cdot\{z\} \tag{17}$$

where e is the ($3n\times1$) elastic, p the ($3n\times1$) plastic part and S is the ($3n\times3n$) stiffness matrix of the structure.

3 LIMIT ANALYSIS WITH MATHEMATICAL PROGRAMMING

The aim of limit analysis is the determination of the maximum load that a structure can sustain with static and kinematic limitations. In this sense, the evaluation of the maximum load can be dealt as a maximization problem with equilibrium, compatibility, yield and complementarity constraints. Thus the system of equations (1), (14), (15), (16) and (17) embedding the yield hyperplane identification and the multi-linear hardening/softening behavior can be converted into the following optimization problem:

$$
\left.\begin{array}{ll}
\text{maximize} & a-\rho\cdot\{w\}^T\cdot\{z\} \\
\text{subject to} & [B]\cdot\{s\}-a\cdot\{f\}=\{f_d\} \\
 & [S]^{-1}\cdot\{s\}-[B]^T\cdot\{u\}+[N]\cdot\{z\}=0 \\
 & \{w\}=-[N]^T\cdot\{s\}+[H]\cdot\{z\}+[H_0]+\{r\}\geq 0,\quad \{z\}\geq 0
\end{array}\right\} \qquad (18)
$$

The solution of the above problem provides simultaneously the load multiplier *a*, the corresponding stresses *s* and displacements *u* together with the plastic multipliers *z*. It is noted that the discrete nature of the complementarity condition is dealt by moving the complementarity term to the objective function and properly penalizing it with an iteratively increasing parameter ρ [12,13,15]. This treatment of the complementarity condition converts the optimization problem into a Non-Linear Programming (NLP) problem that is sensitive to initial values of variables and parameter ρ.

4 NUMERICAL EXAMPLE

The optimization problem described in relation (18) is implemented in Matlab code for the analysis of steel frame structures. The data are processed by *fmincon* solver (appropriate for the minimization of constrained nonlinear multivariable function), with the interior-point algorithm selected as optimization method. The aim is to investigate the role of combined axial-shear force-bending moment interaction and its influence on structural behavior. For this purpose, a steel plane frame has been examined for the following cases:

- Case (a): Limit analysis, rigid-perfectly plastic, NM interaction.
- Case (b): Limit analysis, multi-segment isotropic hardening behavior, NM interaction.
- Case (c): Limit analysis, rigid-perfectly plastic, NQM interaction.
- Case (d): Limit analysis, multi-segment isotropic hardening behavior, NQM interaction.

The example concerns the three-storey, four-bay plane frame shown in Figure 4 that is subjected to increasing lateral and vertical loading. The frame is discretized into 39 elements, 32 nodes and 81 degrees of freedom. The steel grade is S235 with $E = 2\times10^8\ kN/m^2$. Sections HEA300 and IPE330 are employed for all columns and beams respectively. The assumed multi-segment hardening/softening behavior depends on the parameters of every section. More specifically, for columns $h_1=2600kNm$ $z_1=0.005$ $\lambda_1=1.04$, $h_2=1625kNm$ $z_2=0.015$ $\lambda_2=1.10$, $h_3=-1392kNm$ $z_3=0.05$ $\lambda_3=0.935$, while for beam cross-sections $h_1=1260.1kNm$ $z_1=0.003$ $\lambda_1=1.02$, $h_2=810.04kNm$ $z_2=0.01$ $\lambda_2=1.05$, $h_3=-992.3kNm$ $z_3=0.03$ $\lambda_3=0.945$. The initial value of $\rho=1000$ for case (b) and $\rho=100$ for case (d) with an updating rule of $\rho=10\rho$ after each NLP solution until an appropriate convergence tolerance is reached ($w^Tz\leq 10^{-6}$).

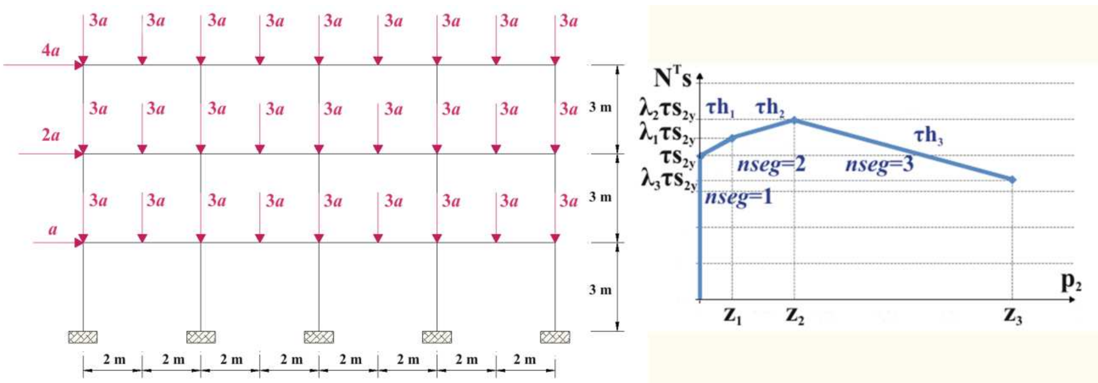

Figure 4: Three-storey, four-bay plane frame and its multi-linear hardening/softening behavior.

	Cases	(a)	(b)	(c)	(d)	(α_{NQM}-α_{NM})/α_{NQM} %	
						P. Plastic	Hardening
Frame	**α** (kN)	**108.33**	**109.42**	**95.29**	**97.11**	**-13.68**	**-12.68**
	top-storey *u* (m)	**0.165**	**0.141**	**0.167**	**0.158**		

Table 2: Analysis results of frame.

It is observed that the load carrying capacity of the structure is significantly reduced for the case of NQM interaction for both structural behaviors. In Figure 5 the ultimate states for all cases are presented. In Figure 5b and 5c the corresponding number of hardening/softening segment is inscribed next to each plastic hinge.

Although the plastic hinge pattern for cases (b) and (d) are identical, there is a difference in the the stress state of plastic hinges. The role of shear force is more evident at column bases for case (d). The interaction diagrams for all cases are depicted in Figure 6. Black and blue spots denote cross sections of *j* and *k* element end respectively. For NM interaction the dominant role of bending moment is evident, while the role of axial force is more intense for beam cross sections, especially for the right ends of first and second storeys that reside at their softening branch. Bending moment maintains its paramount role for NQM interaction, but shear force effect is stronger than that of axial force.

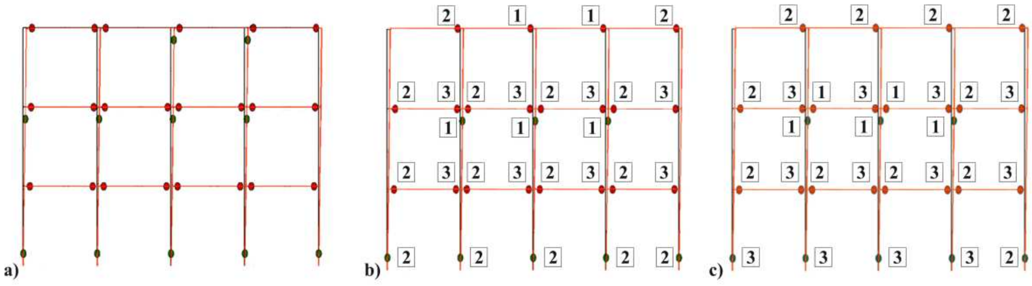

Figure 5: Plastic hinge formation of frame for cases a) (a) and (c), b) (b) and c) (d).

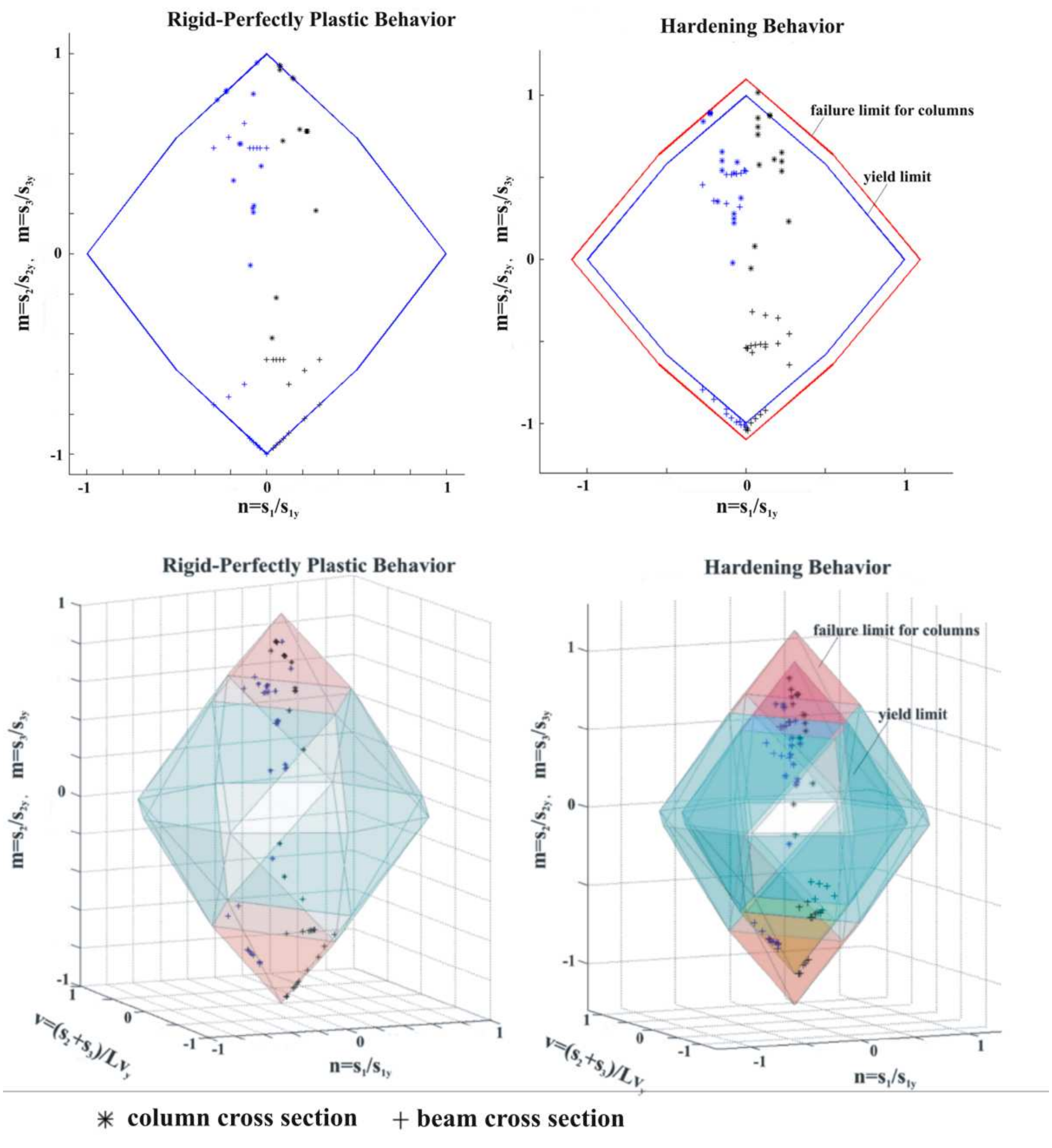

Figure 6: Interaction diagrams for all cases.

5 CONCLUDING REMARKS

The elastoplastic analysis under holonomic behavior and mathematical programming evaluates the maximum load carrying capacity of a structure by solving an optimization problem subjected to constraints that enforce equilibrium, compatibility, yielding and the complementarity conditions. Due to the disjunctive nature of the latter, the aforementioned optimization problem is reformulated as a NLP problem by using a penalty function method which performs satisfactorily in terms of robustness and efficiency.

Herein, the proposed formulation is based on the cone identification that precedes the development of both plastic deformations and multi-segment hardening/softening behavior. Every critical section, at any loading stage, belongs to a specific cone of the interaction diagram targeting only one yield hyperplane. Having this information the yield condition is formed only for this specific hyperplane and not for all existing ones. This reduces considerably the number of yield constraints, decreasing the complexity of the problem which becomes independent of the number of planes that approximate the nonlinear yield surface. Furthermore, incorporation of multi-linear hardening is also simplified and independent of the number of linear segments describing hardening/softening behavior. Having the critical yield hyperplane identified, only one plastic multiplier z is potentially needed for each cross section. As a consequence, hardening matrices are formed only for this plastic multiplier. This results also in drastic reduction of the size of complementarity condition which due to its discrete nature is the source of many inherent numerical problems.

The optimization problem can incorporate various yield criteria in linearized form. In this work, the generalized Gendy-Saleeb yield criterion is adopted that accounts for the axial-shear force-bending moment interaction. Numerical results highlight that shear force effect in yield condition may be significant as it leads to reduction of the load carrying capacity and thus to unsafe design, as compared to axial force-bending moment interaction. Moreover, for more complex configurations this may lead also to different collapse mechanisms.

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