THE ROLE OF REDUNDANCY AND OVERSTRENGTH IN EARTHQUAKE RESISTANT DESIGN

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Abstract: *The role of redundancy and overstrength in frame structures under seismic excitation is investigated. More specifically, the validity of three different redundancy indices introduced in the literature and of an energy based redundancy measure is examined on the basis of inelastic dynamic analysis. These indices are evaluated for 2D reinforced concrete frames with reference to the influence of number of bays, rotational ductility capacity ratios, lifespan and seismic intensity. Moreover, the role of overstrength in the framework of Capacity Design, as specified in current codes, and the importance of its spatial distribution according to the notion of weak beams-strong columns, are discussed. Numerical results that demonstrate the efficiency of existing and proposed measures towards a more safe design are presented and properly interpreted.*

1 INTRODUCTION

Redundancy and overstrength of structures are two distinct notions the synergy of which affects the overall behavior of frame structures. The quantification of both redundancy and overstrength during a seismic excitation is a decisive factor that together with ductility contributes to the assessment of structural seismic response and constitutes the basis of an efficient structural design.

The beneficial effect of both redundancy and overstrength are widely accepted, but they have not been quantified in an objective way, so that they contribute to a more efficient aseismic design. This is due to the variability and uncertainty of the seismic excitation that leads to an uncertain and widely variable structural response (as there are different ways of seismic energy absorption, creation of plastic hinges and exhaustion of overstrength). The European Codes do not address the quantitative influence of redundancy and consider a uniform factor q that encompasses all the parameters of inelastic behavior of structures, while overstrength is taken into account within the framework of Capacity Design. On the contrary, in the US Codes there are factors that quantify separately the redundancy and overstrength of a structure. However, these factors lack of the appropriate generality due to the fact that the influence of both redundancy and overstrength are not fully clarified.

The aim of this work is to elaborate the concepts of redundancy and overstrength, to elucidate their dissociation and to attempt to quantify their influence over the seismic response of reinforced concrete buildings.

2 REDUNDANCY

2.1 Introduction

The first efforts of assessing the influence of redundancy under earthquake loading were made by Frangopol and Curley[2], [3]. They showed that the definition of redundancy as the static indeterminacy of structures is inadequate for the consideration of their seismic response, proposing additional redundancy measures that correlate the strength of damaged and undamaged structural system.

Pandey and Barai[7] presented a generalized definition of redundancy defined in terms of structural response sensitivity, which indicates that the structure can have different degrees of redundancy during its lifetime.

Bertero and Bertero[1] connected the influence of redundancy with failure probability of a structure under seismic excitation. They defined redundancy as the number of plastic hinges n that yield or fail at structural member ends until collapse. Based on this definition, they tried to quantify the effect of redundancy on the failure probability of a structure taking into account the coefficients of variation of loads and strengths, the inherent overstrength and ductility. The effect of earthquake loading was estimated by a static nonlinear pushover analysis. They concluded that the load variation should be decreased relatively to capacity variation in order to take advantage of redundancy based on reliability grounds.

Wang and Wen[10], [11] introduced a uniform-risk redundancy factor R_R, which is the ratio of spectral displacement capacity for incipient collapse over the spectral displacement corresponding to a specified allowable probability of incipient collapse. Wen and Song[12] criticized the validity of the Uniform Building Code ρ factor, which considers only the structural configuration neglecting the uncertainties in loading and strength. Moreover, they compared the strength requirements based on uniform-risk factor method (according to Wang and Wen[10], [11]) and Uniform Building Code ρ factor method and they reached the conclusion that ρ factor overestimates the required strength irrelevantly of structural ductility.

Husain and Tsopelas[4], [8], in their attempt to quantify the effect of redundancy, introduced two indices: a) redundancy-strength index r_s, which expresses the ability of a structural system to redistribute stresses from yielded or failed elements towards elements with higher resistance, and b) redundancy-variation index r_υ, which quantifies the effects of element strength on the structural system strength. These indices were examined in 2D reinforced concrete frames with respect to the number of stories, the number of vertical lines of resistance for various beam ductility capacity ratios by using static nonlinear pushover analysis. They also studied the redundancy response modification factor R_R and the reliability index β and presented simplified expressions for their evaluation using the redundancy indices r_s and r_υ.

In this work, an attempt to quantify the influence of key parameters that account for the redundancy and overstrength in the context of dynamic inelastic analysis is undertaken.

2.2 Measures of structural redundancy

The aforementioned redundancy indices have been examined on the basis of static inelastic pushover analysis. This entails that the dynamic response of structures cannot be fully captured, since important time-dependent parameters of seismic response are not taken into account (i.e. excitation amplitude, frequency content and duration). In this work, dynamic analysis is used to investigate the validity of certain redundancy indices while also an energy based indirect measure of the response is introduced. Inelastic dynamic analysis can ascribe more precisely the features of reinforced concrete structures, e.g. stiffness degradation and strength deterioration under cyclic loading.

The indices examined are:

- The number of plastic hinges (PHs) n that yield or fail at structural member ends until the total collapse (Bertero and Bertero[1]).
- Redundancy-strength index $r_s = \dfrac{\overline{S_u}}{\overline{S_y}}$ (Husain and Tsopelas[8]) where $\overline{S_u}$ is the mean value of ultimate strength and $\overline{S_y}$ is the mean yield strength. This index constitutes a deterministic measure for quantifying redundancy.

It is noted that the redundancy-strength index r_s in dynamic analysis is deprived of its physical meaning because the value of strength at the point of collapse does not represent the ultimate structural strength. For this reason, it is expressed in the following way:

$$r_s' = \frac{V_{\max}}{V_y} = \frac{\ddot{u}_{g\max}}{\ddot{u}_{g,y}} \tag{1}$$

where $V_{\max} = M \cdot \ddot{u}_{g\max}$ is the maximum base shear of the structure, $V_y = M \cdot \ddot{u}_{g,y}$ is the base shear at first yielding, $\ddot{u}_{g\max}$ is the maximum seismic acceleration and $\ddot{u}_{g,y}$ is the seismic acceleration at instant of the first yielding of the structure.

- Redundancy-variation index $r_\upsilon = \sqrt{\dfrac{1+(n-1)\cdot\overline{\rho}}{n}}$ (Husain and Tsopelas 2004) where n represents the number of plastic hinges according to Bertero and Bertero[1] and $\overline{\rho}$ is the average correlation coefficient of the strengths of plastic hinges. This index aims at expressing the probabilistic nature of redundancy.
- Redundancy-energy index r_{en}. This index is introduced herein for the quantification of the overall influence of redundancy over the structural strength through the input kinetic energy of seismic excitation. The redundancy-energy index r_{en} is defined as follows:

$$r_{en} = \frac{E_u}{E_y} \tag{2}$$

where E_u is the total seismic energy for the lifespan of the structure until collapse and E_y is the seismic energy up to the first yielding. The concept is based on the fact that E_y constitutes the seismic energy that would cause collapse of the same structure if it was nonredundant (then $r_{en}=1$). Thus, the additional seismic energy introduced in the structural system is mainly due to its redundancy (its ability to redistribute loading after first yielding). Furthermore, this index corresponds to an indirect measure of structural behavior as stiffness distribution is related to the duration until the initial yield and the distribution of strength within the structure affects the lifespan until collapse. The input seismic energy inserted into the structure is given as:

$$E(t) = 1/2 \cdot M \cdot \int_0^t \dot{u}_g^2(t) \cdot dt \tag{3}$$

Thus the total seismic energy until collapse is given by the following expression:

$$E_u = \frac{1}{2} \cdot M \cdot \int_0^{t_u} \dot{u}_g^2(t) \cdot dt = \frac{1}{2} \cdot M \cdot \Delta t \cdot \sum_{i=1}^{N_u} \dot{u}_g^2(t_i) \tag{4}$$

where $t_u = N_u \cdot \Delta t$ and Δt is the time step of integration.

Similarly, the initiated seismic energy until the first yielding is defined as:

$$E_y = \frac{1}{2} \cdot M \cdot \int_0^{t_y} \dot{u}_g^2(t) \cdot dt = \frac{1}{2} \cdot M \cdot \Delta t \cdot \sum_{i=1}^{N_y} \dot{u}_g^2(t_i) \tag{5}$$

where $t_y = N_y \cdot \Delta t$.

Thus Eq. (1) becomes:

$$r_{en} = \frac{\sum_{i=1}^{N_u} \dot{u}_g^2(t_i)}{\sum_{i=1}^{N_y} \dot{u}_g^2(t_i)} \tag{6}$$

As shown in Fig.1, the estimation of the above ratio for different structures subjected to the same excitation is marked on a single plot together with fist yielding and collapse instances for every structure that characterize the duration of the elastic and inelastic response respectively.

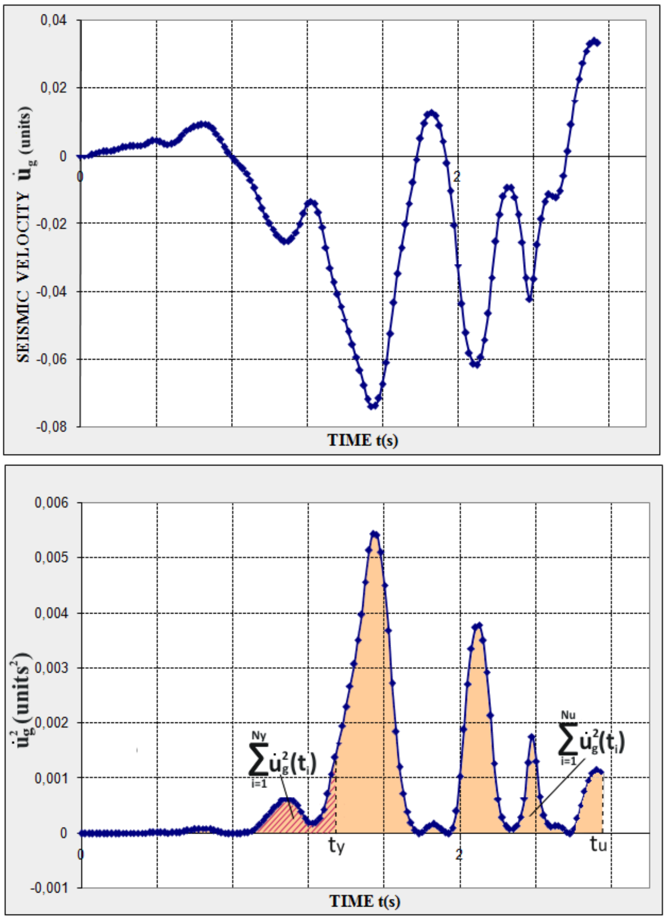

Figure 1. Calculation of redundancy-energy index r_{en}.

2.3 Variation of redundancy indices under dynamic analysis

The effects of the aforementioned redundancy indices are examined for various 2D reinforced concrete 3-storey frames with different number of bays, following an inelastic dynamic analysis until collapse, monitoring rotational ductility capacity μ_θ, lifespan and the effect of seismic intensity. The analyses are performed using "Plastique" (Koumousis, Chatzi and Triantafyllou[5]), i.e. a computer code based on spread plasticity macroelements following a Bouc-Wen type of hysteretic behavior with stiffness degradation and strength deterioration, properly modified to assess the redundancy indices under examination.

In Table 1 the properties for all 2D frames are given, while in Table 2 the features of the scaled El Centro seismic excitation are presented. The different types of frames analyzed are shown in Fig.2. The different frames are marked with three subscripts as: *Fx.y.j.*, where x is the number of bays, y is the number of stories and j is the number of frame referring to different properties.

COLUMNS DIMENSIONS (m)	0,25 × 0,25
BEAMS DIMENSIONS (m)	0,20 × 0,30
LENGTH OF BAYS (m)	4,5
HEIGHT OF STORIES (m)	3,0

Table 1. Geometrical characteristics of plane frames.

EARTHQUAKE	EL CENTRO
EARTHQUAKE SCALE	2
DURATION OF EXCITATION	20 s
TIME STEP Δt	0,02 s
MAXIMUM GROUND ACCELERATION	6,26 m/s^2

Table 2. Features of common seismic excitation.

Figure 2. Different types of 2D frames

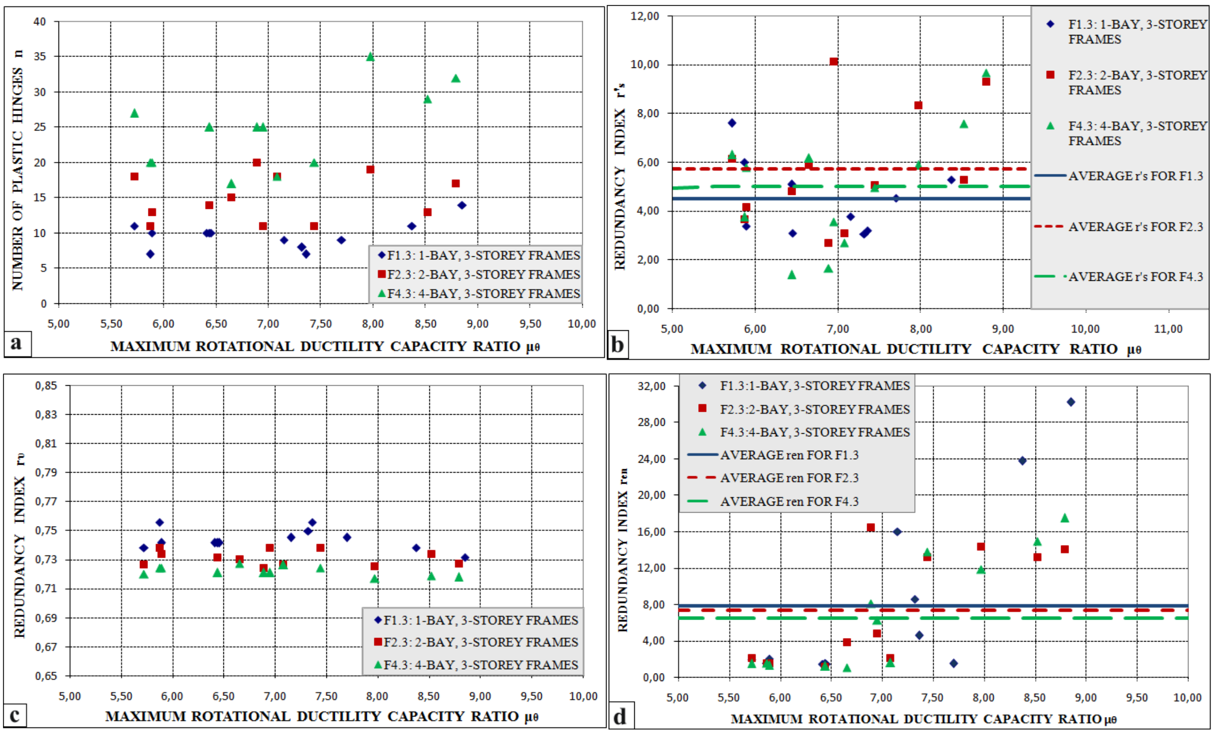

Figure 3. Redundancy indices versus maximum rotational ductility capacity ratio μ_θ for frames with different number of bays.

In Fig. 3 the effect of ductility and number of bays on the chosen redundancy indices is presented. In Fig.3a it is evident that for the same values of maximum rotational ductility capacity μ_θ the frames with more bays form more plastic hinges. In Fig.3b the revised redundancy index r'_s is plotted as a function of maximum rotational ductility capacity μ_θ. There is no apparent relation between r'_s and μ_θ. This stems from the fact that all frames present the same value of $\ddot{u}_{g\max}$ since the seismic excitation is common, hence r'_s results in representing a measure of $\ddot{u}_{g,y}$ that is time-dependent and does not represent the total structural behavior until the first yielding. The variance of redundancy-variation index r_υ towards maximum μ_θ is illustrated in Fig.3c. It is worth noting that the value of the average correlation coefficient $\overline{\rho}$ has been chosen arbitrary ($\overline{\rho} = 0.5$) without affecting the generality of conclusions, since the interest is focused on the relative effects of redundancy variation index r_υ as a function of rotational ductility and number of bays. It is observed that r_υ decreases as the number of bays increases, namely the probabilistic effects of redundancy are greater for frames with more bays. This is reasonable because the values of r_υ are inversely proportional to the number of plastic hinges at failure and frames with more bays form more PHs. In the following, Fig.3d depicts the redundancy-energy index r_{en} versus maximum rotational ductility μ_θ. A clearly marked tendency of increase, especially for great values of maximum μ_θ, is ascertained. As far as the average values of r_{en} are concerned, one-bay frames present the greatest values, followed by two-bay frames and lastly four-bay frames. The values of this index are directly related to the lifespan of different frames. Moreover, there is a correspondence between mean values of $\overline{r_{en}}$ and mean values of lifespan $\overline{Ls}$ of the frames, i.e. frames with greater mean values of $\overline{r_{en}}$ present also greater mean values of $\overline{Ls}$ ($\overline{Ls}_{F1.3} = 1{,}91s$, $\overline{Ls}_{F2.3} = 1{,}54s$ and $\overline{Ls}_{F4.3} = 0{,}73s$).

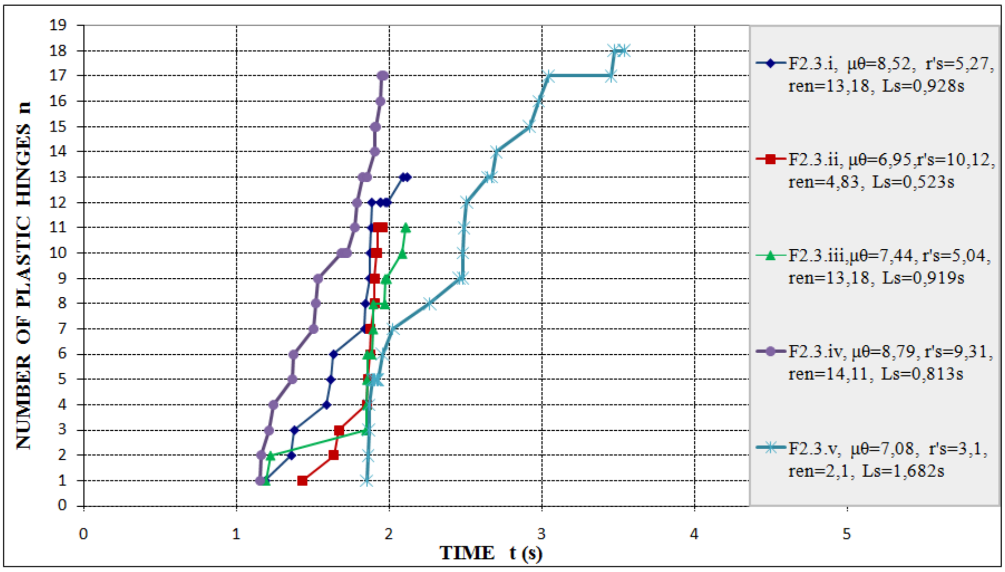

Figure 4. Evolution of number of PHs for 2-bay, 3-storey frames.

The evolution of structural redundancy is presented in Fig.4. It is concluded that frames with the same number of PHs n (F2.3.ii and F2.3.iii) consume different amounts of seismic energy (different values of r_{en}) and have different lifespan Ls. Indeed F2.3.iii frame that presented a greater value of r_{en} compared to F2.3.ii frame survived also longer. Furthermore, it is ascertained that frames with the same values of r_{en} and for common time start of yielding (F2.3.i and F2.3.iii) present also similar lifespan ($Ls_{F2.3.i} = 0{,}928s$ and $Ls_{F2.3.iii} = 0{,}919s$). Between these two frames the one that has formed more PHs is that with the greatest value of maximum μ_θ. The F2.3.iv frame presented the greatest lifespan Ls and n. This is directly related to the fact that this frame started yielding later than the others and consequently consumed less seismic energy.

In Fig.5 the effect of seismic intensity on the evolution of plastic hinges is presented. The greater the seismic intensity is, the steeper the slope of the number of formed hinges is, followed by smaller the values of Ls. Moreover, it is observed that the lines are shifted towards smaller values of time as the seismic scale increases, since yielding occurs earlier. It is also evident that the values of r_{en} present an expected increase as the seismic scale increases since frames have to consume larger amounts of seismic energy. The number of PHs n and the lifespan Ls generally decrease as the seismic intensity increases. However, there are some exceptions such as the F2.3.v frame that formed more PHs and F2.3.ii frame that lasted longer for more intensive seismic excitation. This accrues from the fact that different load redistributions occur at each frame with respect to variation of seismic input and lead to different sequences of plastic hinges and consequently to different lifespan.

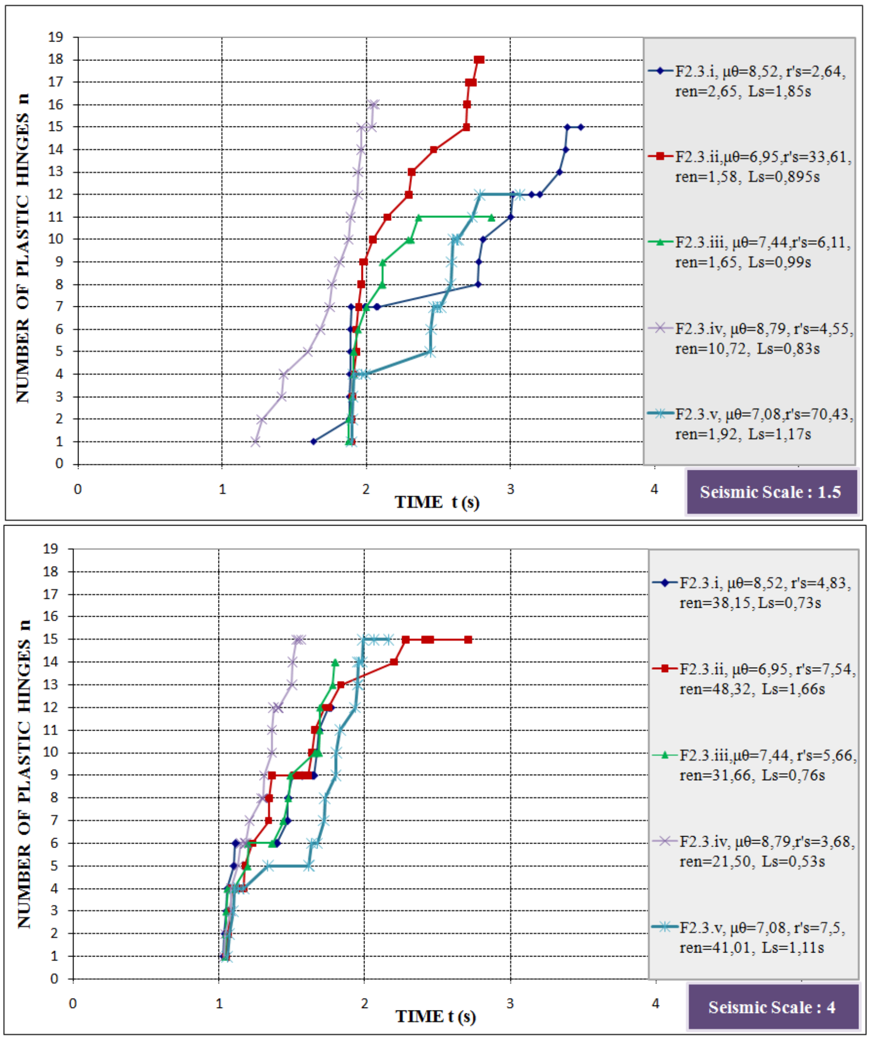

Figure 5. Effect of seismic intensity on evolution of PHs.

3 OVERSTRENGTH

Previous investigation on performance of buildings during severe earthquakes indicates that structural overstrength plays a very important role in protecting buildings from collapse. Quantification of actual overstrength can be employed to reduce the forces used in the design, hence leading to more economical structures. The main sources of overstrength are reviewed in other studies (Uang[9], Mitchell and Paulter[6]). These are attributed to: the difference between the actual and the design material strength; conservatism of the design procedure and ductility requirements; load factors and multiple load cases; accidental torsion consideration; serviceability limit state provisions; participation of nonstructural elements; effect of structural elements not considered in predicting the lateral load capacity (e.g. actual slab width); minimum reinforcement and member sizes that exceed the design requirements; redundancy; strain hardening; actual confinement effect; and utilizing the elastic period to obtain the design forces.

In Greek Earthquake Resistant Design Code[13] the distribution of overstrength in a building is guided through the weak beam-strong column concept. This is aimed at avoiding formation of soft-storey mechanism, or local column collapse. Assuming all beams and columns having different strength, it turns out that, if a weak beam–strong column concept is followed, the maximum load depends on the beam strengths and the strength of the lower column at its bottom. This leads to the lateral collapse mechanism, provided that stiffness distribution is such that does not alter significantly the distribution of stresses. However, this widely accepted principle does not apply to more realistic cases where critical sections behave following a bilinear law and/or there exist an interaction between stress resultants. In these cases, yielding occurs at beam sections first, but column sections may fail (reach their ultimate strength) before beam failure in a weak beam-strong column designed joint.

At this part the validity of the principles of Capacity Design concerning overstrength distribution is examined. For this purpose 2D reinforced concrete frames with various bays of equal spans and various stories of equal height designed according to the Greek Earthquake Resistant Design Code[13] are examined. Their design is based on NEXT program following an elastic analysis, whereas “Plastique”[5] was used for the subsequent inelastic dynamic analysis. The El Centro earthquake scaled by a factor of 4 is utilized to ensure the collapse of all frames. The results of dynamic analyses are presented in Fig.6. The illustrated sequence of PH formation of the F3.3 frame presented in Fig.6a fully verifies the principles of Capacity Design. However, in case of frames with more stories, the objectives of earthquake resistant design are violated, as shown in Fig.6b. In this, there is a column that fails before the beams at the same joint and, as a consequence, the structure is led to premature collapse. This fact entails that the principles of capacity design ensure the desirable sequence of PHs only at initial circles of loading-unloading-reloading, whereas, afterwards, due to the change of structural system and the intensive load redistributions, the violation of constraints of Capacity Design is also possible. In addition, Capacity Design concept is of local character (control at joint level), unable of control the pace of the entire structure towards collapse. Therefore, in cases of structures of more complicated connectivity there is no guarantee that a column will fail prior to the beam that form the same joint.

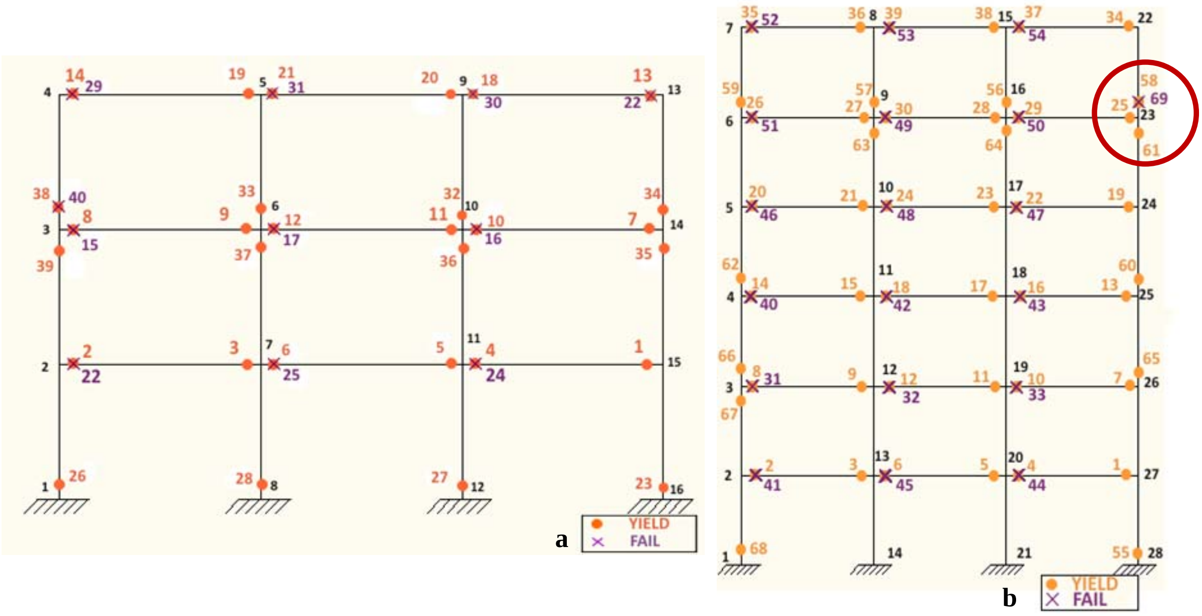

Figure 6. Evolution of PH formation in reinforced frames.

4 CONCLUDING REMARKS

In this work the quantification of structural redundancy and the principles of distribution of overstrength have been examined for various 2D reinforced concrete frames under seismic excitation.

The main remarks drawn on the basis of numerical investigation of structural redundancy can be summarized as follows:

- Dynamic analysis gives different results as compared to pushover analysis for the same redundancy indices with respect to the same parameters. Generally, an increasing tendency of all redundancy indices with respect to maximum rotational ductility μ_θ is observed, but this tendency is not as apparent as in the case of pushover analysis.
- The number of plastic hinges (PHs) n that yield or fail at structural member ends until the total collapse (Bertero and Bertero 1999) constitutes an adequate measure of utilized structural redundancy, but it does not encompass information for other important parameters such as time or initial redundancy. Therefore n attributes quantitatively the redundancy of a given structure, but it cannot constitute a comparative measure between different structures.
- The index r'_s turns out to be an indication of yield base-shear rather than an index of redundancy.
- The redundancy-variation index r_υ (Husain and Tsopelas[4]) captures adequately the probabilistic effects of redundancy on strength of structural systems also in case of dynamic analysis.
- The introduced redundancy-energy index r_{en} constitutes a satisfying measure of redundancy on the grounds that the overall structural behavior is considered. Furthermore, it includes the parameter of time providing indirectly information about the lifespan of the structure.
- The gradient of lines of n-t graphs indicates the rhythm of utilization of redundancy. The grater the gradient is, the faster structural redundancy is exhausted.

- The seismic intensity affects structural redundancy and the duration of exhausting its reserves. The gradient of *n-t* lines becomes steeper and yielding starts earlier as seismic intensity increases. In addition, the number of plastic hinges *n* and lifespan *Ls* appear on the average to decrease for larger values of ground acceleration.

As far as overstrength is concerned, the validity of principles of Capacity Design during seismic excitation is investigated. Various plane reinforced concrete frames are analyzed and the sequences of plastic hinge formation are examined. The desirable sequence of PHs is ensured only at initial circles of loading-unloading-reloading (mainly at yielding phase), while, at later stages, due to intensive load redistributions in the structural system the violation of constraints of Capacity Design are manifested and failure of a column occurs without the former exhaustion of strength reserves of beams.

The quantitative effect of redundancy and overstrength on structural seismic response based on dynamic analysis is examined. Estimation of redundancy is complex due to its interaction with a great number of factors. An energy index offers significant information for the overall behavior. Furthermore, it becomes evident that a more efficient way of overstrength distribution is required that will ensure the desirable sequence of PHs during the entire duration of loading escalated till collapse.

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